Dear students, first of all I would like to wish you Best of Luck!!!!

Set 1

Model Question

Set 2

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. Define linear transformation with an example. Check the following transformation is linear or not? T: $R^2 \to R^2$ be defined by T(x,y) = (x, 2y). Also, let T(x,y) = (3x + y, 5x + 7y, x + 3y). Show that T is a one- to one linear transformation. Does T maps R^2 onto R^3? [3 + 2 + 5]

2. Find the LU factorization of: $\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$.

3. When is a square matrix A said to be diagonalizable? Diagonalize the following matrix, if possible. $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. [2 + 8]

4. Find a least square solution of the inconsistent system Ax = b for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Find an equation involving g, h and k that makes the Augmented matrix correspond to the consistent system:

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right]$$

6. Find the eigen values of matrix $A = \begin{bmatrix} 0.5 & -0.6 \\ 0.75 & 1.1 \end{bmatrix}$ and find basis for each eigen space.

7. Find the dimension of the subspace H= $\left\{ \begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} : a,b,c,d, in, \Re \right\}$.

8. Compute the multiplication of partitioned matrices for A = $\left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & -2 & 7 & -1 \end{array}\right]$

and B= $\left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right]$.

9. The set S = $\{u_1, u_2, u_3\}$ where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ *and* $u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$ is an orthogonal basis for $\Re^3$. Express the vector $y = \begin{bmatrix} 6 \\ 1 \\ -8 \end{bmatrix}$ as a linear combination of the vectors in S.

10. Let A = $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and define T: $R^2 \to R^2$ be defined by T(x) = Ax, find the image under T of $u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ and $u = \begin{bmatrix} a \\ b \end{bmatrix}$.

11. Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}.$$

12. Let $b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$ and $b_3 = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$ and $x = \begin{bmatrix} -8 \\ 2 \\ 3 \end{bmatrix}$.

(a) Show that the set $B=\{ b_1, b_2, b_3 \}$ is a basis for $\Re^3$.

(b) Find the change of co-ordinates matrix from B to the standard basis.

(c) Write the equation that relates x in $\Re^3$ to $[x]_B$.

13. Solve the Leontief production equation for an economy with three sectors given that:

$$A = \begin{bmatrix} 0.2 & 0.2 & 0.0 \\ 0.3 & 0.1 & 0.3 \\ 0.1 & 0.0 & 0.2 \end{bmatrix} \text{ and } x = \begin{bmatrix} 40 \\ 60 \\ 80 \end{bmatrix}.$$

14. Let * be defined on Q^+ by $a * b = \frac{a.b}{3}$. Then show that Q^+ forms a group.

15. Define ring with an example. Compute the product in the given ring (21)(14) in Z_{12} .

The End

Set 3

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. What do you mean by row echelon form and reduced row echelon form? Apply elementary row operation to transform the following matrix first into echelon form and then into reduced echelon form:

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

2. Find the LU factorization of: $\begin{bmatrix} 2 & -4 & -2 & 3 \\ 6 & -9 & -5 & 8 \\ 2 & -7 & -3 & 9 \\ 4 & -2 & -2 & -1 \\ -6 & 3 & 3 & 4 \end{bmatrix}$.

3. Define invertible matrix. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$ if exists.

4. Reduce the matrix A into B using row operation, then

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 3 & 9 & -12 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. Find rank A and dim Nul A

b. Find bases for Col A and Row A.
c. What is the next step to perform to find a basis for Nul A?
d. How many pivot columns are in a row echelon form of A^T ?

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine if the following system is consistent:

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 5x_1 - 8x_2 + 7x_3 &= 1 \end{aligned}$$

6. Define Characteristic equation of matrix. Find the Characteristic equation of matrix $\begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$.

7. Use partitioned matrices to show that $M^2 = I$ when M = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & 1 \end{bmatrix}$.

8. Determine if $\vec{b}$ is a linear combination of vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ where $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$,

$\vec{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$.

9. The set B = $\{1+t, 1+t^2, t+t^2\}$ is a basis for P_2 .Find the co-ordinate vector of p(t) = $6+3t-t^2$ relative to B.

10. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, and a transformation $T: R^2 \mapsto R^3$ defined by T(x) = Ax,

a. Find T(u).
b. Find an x in whose image under T is b.

11. Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & -3 & 1 & -2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{bmatrix}$$ in as few steps as possible.

12. Find a matrix such that W = col A where W = $\left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b, ...in.....\Re \right\}$.

13. Solve the Leontief production equation for an economy with three sectors given that:

$$A = \begin{bmatrix} 0.0 & 0.5 \\ 0.6 & 0.2 \end{bmatrix} \text{ and } x = \begin{bmatrix} 50 \\ 30 \end{bmatrix}.$$

14. Define Abelian group. Prove that $G = \{-1, 1, i, -i\}$ is an Abelian group.

15. Define ring with an example. Compute the product in the given ring (2, 9).(7, 4) in Z_3 x Z_5.

The End

Set 4

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and a transformation

   $T : R^2 \mapsto R^3$ defined by T(x) = Ax,

   a. Find T(u).

   b. Find an x in whose image under T is b.

   c. Is there more than one x whose image under T is b.

   d. Determine if c is the in the range of the transformation T

2. Define block upper triangular . Assume that A11 is pXp, A22 is qXq and A is invertible .Write a formula for A^{-1} .And hence find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 5 & 6 \end{bmatrix}.$$

3. Find a least squares solution of Ax = b for:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Also determine the least square error for the above least square solution.

4. Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points (2 , 1) , (5 , 2) , (7 , 3) , and (8 , 3).

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. For what value of h and k is the following system consistent: $2x_1 - x_2 = h$ and $-6x_1 + 3x_2 = k$.

6. Solve the system $3x_1 + 4x_2 = 10$, $5x_1 + 6x_2 = 16$ by using the inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

7. If $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$,what value(s) of k ,if any,will make AB = BA ?

8. Let A be a 4x4 matrix and let x be a vector in $\Re^4$. What is the fastest way to compute $A^2 x$?? Count the multiplication.

9. Show that 7 is an Eigen value of the matrix $\begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$? And find the corresponding Eigen vectors.

10. Find the bases for the row space,the column space and the null space of the matrix:

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

11. Prove that the null space of an mXn matrix A is a subspace of $\Re^n$.

12. Define matrix equation of a system of linear equations.

Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$. Does span{ v_1, v_2, v_3 } span $\Re^4$?

If yes find the weights.

13. Determine if $\vec{b}$ is a linear combination of vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ where $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$,

$\vec{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$.

14. Define cyclic group. Prove that $G = \{1, \omega, \omega^2\}$ is a cyclic group.

15. Prove that ($\Re$, +, .) is a ring. Solve the equation $x^2 - 7x + 12 = 0$ in Z_{12} .

The End